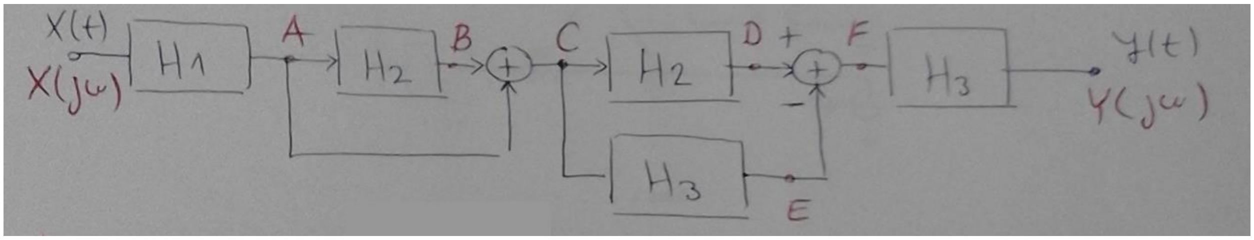
Računske vežbe – Osnovi telekomunikacija – Termin 6

Zadatak 1. Odrediti prenosnu funkciju sistema koji je dat na slici 1. Prenosne funkcije pojedinih sklopova su: $H_1(j\omega)=1/j\omega$, $H_2(j\omega)=e^{-j\omega}$, $H_3(j\omega)=j\omega$.



Slika 1.

Rešenje:

$$X_A(j\omega) = H_1(j\omega)\cdot X(j\omega)$$

$$X_B(j\omega) = H_2(j\omega)\cdot X_A(j\omega) = H_2(j\omega)\cdot H_1(j\omega)X(j\omega)$$

$$X_C(j\omega) = X_B(j\omega) + X_A(j\omega) = H_2(j\omega)\cdot H_1(j\omega)X(j\omega) + H_1(j\omega)X(j\omega) =$$

$$= H_1(j\omega)X(j\omega)\cdot(H_2(j\omega)+1)$$

$$X_D(j\omega) = H_2(j\omega)\cdot X_C(j\omega) = H_2(j\omega)\cdot H_1(j\omega)X(j\omega)(H_2(j\omega)+1)$$

$$X_E(j\omega) = H_3(j\omega)\cdot X_C(j\omega) = H_3(j\omega)\cdot H_1(j\omega)X(j\omega)(H_2(j\omega)+1)$$

$$X_F(j\omega) = X_D(j\omega) - X_E(j\omega) = H_2(j\omega)H_1(j\omega)X(j\omega)(H_2(j\omega)+1) - H_3(j\omega)\cdot H_1(j\omega)X(j\omega)(H_2(j\omega)+1) = \longrightarrow$$

$$= \underbrace{H_1(j\omega)X(j\omega)(H_2(j\omega)+1)}_{\text{ZAJEDNIČKI}}(H_2(j\omega) - H_3(j\omega))$$

$$Y(j\omega) = H_3(j\omega)\cdot X_F(j\omega) = H_3(j\omega)\cdot H_1(j\omega)\cdot X(j\omega)(H_2(j\omega)+1)(H_2(j\omega)-H_3(j\omega))$$

PRENOSNA FUNKCIJA SISTEMA JE

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = H_3(j\omega)\cdot H_1(j\omega)\cdot(H_2(j\omega)+1)(H_2(j\omega)-H_3(j\omega))$$

$$= \cancel{j\omega}\cdot\frac{1}{\cancel{j\omega}}\cdot(e^{-j\omega}+1)(e^{-j\omega}-j\omega) =$$

$$= \underline{\underline{e^{-j2\omega} - j\omega e^{-j\omega} + e^{-j\omega} - j\omega}}$$

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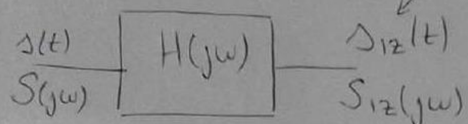
Zadatak 2. Signal *s(t)* dolazi sa prenosnom funkcijom H(jω)=1+*a*Cos(ωt$_1$). Pronaći signal na izlazu sistema. Parametar *a* iznosi 0,4.

Rešenje:

REŠENJE:



POTREBNO JE DA NAĐEMO OVAJ SIGNAL NA IZLAZU IZ SISTEMA

H(jω) MOŽEMO DA IZRAZIMO KAO: (JER JE $\cos x = \frac{e^{jx}+e^{-jx}}{2}$)

$$H(j\omega) = 1 + a\,\frac{e^{j\omega t_1}+e^{-j\omega t_1}}{2} = 1+\frac{a}{2}e^{j\omega t_1}+\frac{a}{2}e^{-j\omega t_1}$$

SPEKTAR NA IZLAZU SISTEMA SE IZRAŽAVA PREKO:

$$S_{iz}(j\omega) = H(j\omega)\cdot S(j\omega)$$

PRENOSNA FUNKCIJA SISTEMA — SPEKTAR NA ULAZU SISTEMA

POŠTO JE POTREBNO DA NAĐEMO SIGNAL U VREMENSKOM DOMENU NA IZLAZU SISTEMA, PIŠEMO IZRAZ ZA FT:

$$s_{iz}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{iz}(j\omega)\,e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(j\omega)\cdot S(j\omega)\cdot e^{j\omega t}\,d\omega =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} S(j\omega)\cdot\left(1+\frac{a}{2}e^{j\omega t_1}+\frac{a}{2}e^{-j\omega t_1}\right)e^{j\omega t}\,d\omega = \Big/\begin{matrix}\text{IZMNOŽIMO}\\ \text{ZAGRADU}\end{matrix}\Big/ =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} S(j\omega)\,e^{j\omega t}\,d\omega + \frac{1}{2\pi}\int_{-\infty}^{+\infty} S(j\omega)\cdot\frac{a}{2}e^{j\omega t_1}e^{j\omega t}\,d\omega + \frac{1}{2\pi}\int_{-\infty}^{+\infty} S(j\omega)\cdot\frac{a}{2}e^{-j\omega t_1}\cdot e^{j\omega t}\,d\omega =$$

$$= \underbrace{\frac{1}{2\pi}\int_{-\infty}^{+\infty} S(j\omega)\,e^{j\omega t}\,d\omega}_{s(t)} + \frac{a}{2}\underbrace{\frac{1}{2\pi}\int_{-\infty}^{+\infty} S(j\omega)\cdot e^{j\omega(t_1+t)}\,d\omega}_{s(t+t_1)} + \frac{a}{2}\cdot\underbrace{\frac{1}{2\pi}\int_{-\infty}^{+\infty} S(j\omega)\cdot e^{j\omega(t-t_1)}\,d\omega}_{s(t-t_1)}$$

NA OSNOVU PRETHODNOG ZAKLJUČUJEMO DA JE SIGNAL NA IZLAZU SISTEMA JEDNAK:

$$s_{iz}(t) = s(t) + \frac{a}{2}s(t+t_1) + \frac{a}{2}s(t-t_1) = \Big/\begin{matrix}\text{ZAMENIMO}\\ \text{VREDNOST } a=0.4\end{matrix}\Big/ =$$

$$= s(t) + 0.2 * s(t+t_1) + 0.2 * s(t-t_1)$$

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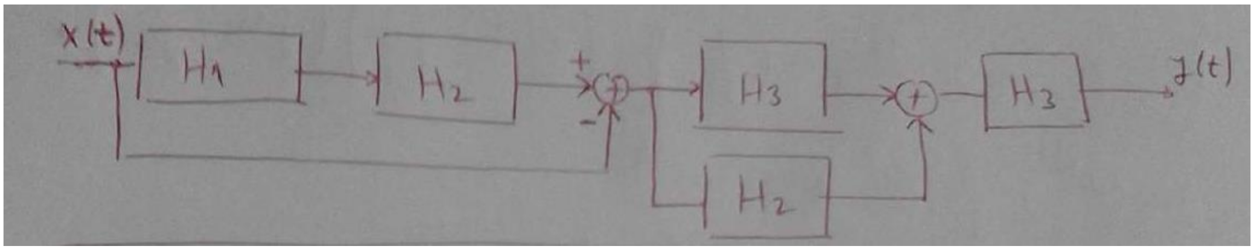
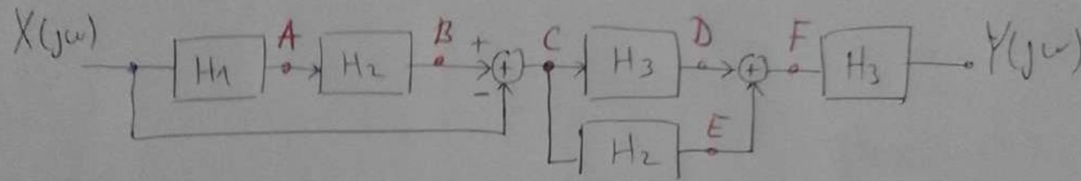
Zadatak 3. Na slici je prikazana struktura nekog linearnog sistema. Prenosne funkcije pojedinih skolopova su: $H_1(j\omega) = j\omega$, $H_2(j\omega) = e^{j\omega}$ i $H_3(j\omega) = e^{-j\omega}$.

a) Odrediti prenosnu funkciju sistema.
b) Ako na ulazu sistema deluje impuls $x(t) = \begin{cases} t, |t| \le 1 \\ 0, |t| > 1 \end{cases}$ skicirati signal na izlazu sistema.
c) Pronaći vrednost signala na izlazu u tački t=-0.5



Rešenje:

Prelazimo u domen frekvencije, crtamo novu sliku, obeležimo sve neobeležene linije i tražimo izlaze iz sklopova počev od ulaza



$$X_A(j\omega) = H_1(j\omega)\cdot X(j\omega)$$

$$X_B(j\omega) = H_2(j\omega)\cdot X_A(j\omega) = H_2(j\omega)\cdot H_1(j\omega)\cdot X(j\omega)$$

$$X_C(j\omega) = X_B(j\omega) - X(j\omega) = H_2(j\omega)\cdot H_1(j\omega)\cdot X(j\omega) - X(j\omega) = X(j\omega)\left(H_2(j\omega)H_1(j\omega)-1\right)$$

$$X_D(j\omega) = H_3(j\omega)\cdot X_C(j\omega) = H_3(j\omega)\cdot X(j\omega)\cdot\left(H_2(j\omega)\cdot H_1(j\omega)-1\right)$$

$$X_E(j\omega) = H_2(j\omega)\cdot X_C(j\omega) = H_2(j\omega)\cdot X(j\omega)\left(H_2(j\omega)H_1(j\omega)-1\right)$$

$$X_F(j\omega) = X_D(j\omega) + X_E(j\omega) = H_3(j\omega)\cdot X(j\omega)\cdot\left(H_2(j\omega)H_1(j\omega)-1\right) +$$
$$+ H_2(j\omega)\cdot X(j\omega)\cdot\left(H_2(j\omega)H_1(j\omega)-1\right) =$$
$$= X(j\omega)\left(H_2(j\omega)\cdot H_1(j\omega)-1\right)\left(H_3(j\omega)+H_2(j\omega)\right)$$

$$Y(j\omega) = H_3(j\omega)\cdot X_F(j\omega) = H_3(j\omega)\cdot X(j\omega)\left(H_2(j\omega)\cdot H_1(j\omega)-1\right)\left(H_3(j\omega)+H_2(j\omega)\right)$$

NAĐEMO PRENOSNU FUNKCIJU SISTEMA KAO ODNOS SPEKTRA NA IZLAZU I ULAZU U SISTEM:

$$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{H_3(j\omega)\cdot\cancel{X(j\omega)}\left(H_2(j\omega)\cdot H_1(j\omega)-1\right)\left(H_3(j\omega)+H_2(j\omega)\right)}{\cancel{X(j\omega)}}=$$

$$H(j\omega)=H_3(j\omega)\cdot\left(H_2(j\omega)H_1(j\omega)-1\right)\left(H_3(j\omega)+H_2(j\omega)\right)$$

PA ZAMENIMO VREDNOSTI PRENOSNIH FUNKCIJA ZA POJEDINE SKLOPOVE

$$H(j\omega)=e^{-j\omega}\cdot\left(e^{j\omega}\cdot j\omega-1\right)\cdot\left(e^{-j\omega}+e^{j\omega}\right)=$$

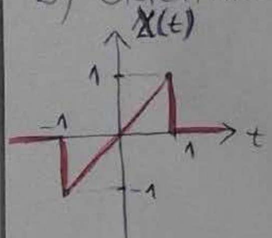
$$=\left(\cancel{e^{-j\omega}\cdot e^{j\omega}}\cdot j\omega-e^{-j\omega}\right)\cdot\left(e^{-j\omega}+e^{j\omega}\right)=$$

$$=\left(j\omega-e^{-j\omega}\right)\left(e^{-j\omega}+e^{j\omega}\right)=j\omega\cdot e^{-j\omega}+j\omega e^{j\omega}-e^{-j2\omega}-e^{0}=$$

$$=j\omega\cdot e^{-j\omega}+j\omega\cdot e^{j\omega}-e^{-j2\omega}-1$$

W

b) SKICIRAMO SIGNAL NA ULAZU SISTEMA:



$$Y(j\omega)=H(j\omega)\cdot X(j\omega)$$

$$Y(j\omega)=\left(j\omega\cdot e^{-j\omega}+j\omega\cdot e^{j\omega}-e^{-2j\omega}-1\right)\cdot X(j\omega)$$

$$Y(j\omega)=j\omega\cdot e^{-j\omega}X(j\omega)+j\omega\cdot e^{j\omega}X(j\omega)-e^{-j2\omega}X(j\omega)-X(j\omega)$$

PRIMENIMO

$$s(t)\rightarrow S(j\omega)$$
$$s(t+T_0)\rightarrow e^{j\omega T_0}S(j\omega)$$
$$\frac{d\,s(t+T_0)}{dt}\rightarrow j\omega\cdot e^{j\omega T_0}S(j\omega)$$

$$Y(j\omega)\rightarrow y(t) \qquad X(j\omega)\rightarrow x(t)$$

$$j\omega\cdot e^{-j\omega}X(j\omega)\rightarrow\frac{dx(t-1)}{dt}$$

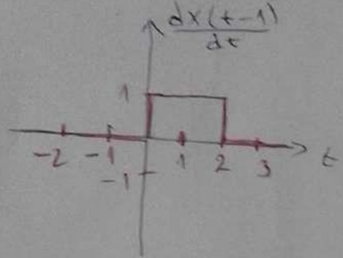
$$j\omega\cdot e^{j\omega}X(j\omega)\rightarrow\frac{dx(t+1)}{dt}$$

$$e^{-j2\omega}X(j\omega)\rightarrow x(t-2)$$

ZAMENIMO U GORNJEM IZRAZU

$$y(t) = \frac{dx(t-1)}{dt} + \frac{dx(t+1)}{dt} - x(t-2) - x(t)$$

SLEDEĆI KORAK JE DA SVAKU FUNKCIJU PREDSTAVIMO GRAFIČKI:



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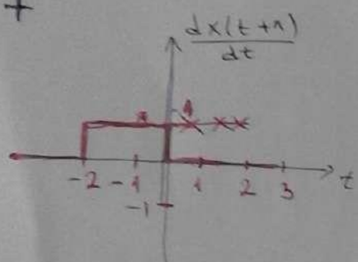

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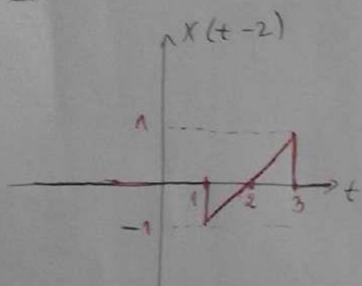

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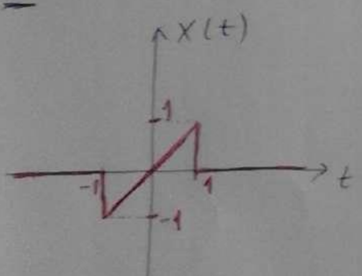
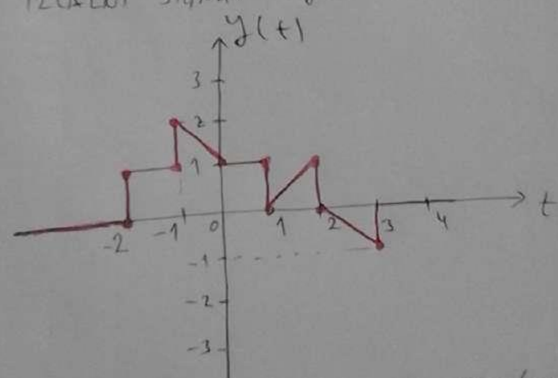


FORMIRAMO TABELU KOJA SADRŽI TAČKE NA t OSI ZA KOJE OVE ČETIRI FUNKCIJE SKOKOVITO MENJAJU VREDNOST

t	$\frac{dx(t-1)}{dt} + \frac{dx(t+1)}{dt} - x(t-2) - x(t) = y(t)$
-2	$0 + 0 - 0 - 0 = 0$
	$0 + 1 - 0 - 0 = 1$
-1	$0 + 1 - 0 - 0 = 1$
	$0 + 1 - 0 - (-1) = 2$
0	$0 + 1 - 0 - 0 = 1$
	$1 + 0 - 0 - 0 \quad 1$
1	$1 + 1 - 0 - 1 = 1$
	$1 + 0 - 1 - 0 = 0$
2	$1 + 0 - 0 - 0 = 1$
	$0 + 0 - 0 - 0 = 0$
3	$0 + 0 - 1 - 0 = -1$
	$0 + 0 - 0 - 0 \quad 0$

NA OSNOVU TABELE NACRTAMO GRAFIK, tj. IZLAZNI SIGNAL $y(t)$



c) POŠTO TRAŽIMO VREDNOST $y(t)$ ZA TAČKU $t=-0.5$, POGLEDAMO SA GRAFIKA U KOM DELU SE NALAZI TA TAČKA I NAPIŠEMO JEDNAČINU PRAVE $y(t)$ U **TOM** DELU:

$$\boxed{y(t) = 1 - \frac{1}{1}t = 1 - t}$$

$$y(-0.5) = 1 - (-0.5) = \underline{1.5\ W}$$